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Knowledge Construction and Collective Practice: At the Intersection of Learning, Talk, and Social Configurations in a Computer-Mediated Mathematics Classroom

Noel Enyedy
Graduate School of Education & Information Studies
University of California, Los Angeles

In this study I investigated how students' mathematical activities, and thereby their mathematical understandings, change as a function of their participation in different social configurations. I examined how the interplay between 2 social configurations—local investigations at a computer simulation and whole-class discussions—contributes to how 7th-grade students learn probabilistic reasoning. I used 2 case studies to investigate (a) how different forms of participation are linked to different social configurations, and (b) how specific discourse practices and ways of reasoning propagate across the classroom and are adopted by individual students. The analyses suggest that classroom mathematical practices are developed, in part, for the social or communicative purpose of settling disputes and not purely for their rational or cognitive value to individuals. Results also provide insight into how to design and orchestrate classroom practice, particularly computer-mediated inquiry, to foster individual learning that is situated within a classroom community oriented toward the construction of a shared understanding of probability.

The learning sciences are just beginning to address the intersection of individual conceptual development and the development of collective, community practices. Recent theoretical work supports the view that developing a deep conceptual un-

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Correspondence and requests for reprints should be sent to Noel Enyedy, University of California, Los Angeles, Graduate School of Education & Information Studies, 2323 Moore Hall, Box 951521, Los Angeles, CA 90095–1521. E-mail: enyedy@gseis.ucla.edu

derstanding of discipline-specific concepts is tied to participation in the discourse practices of disciplinary communities (Greeno & Hall, 1997; Hall, 1999; Lave & Wenger, 1991; Roth, 2001). This theoretical perspective has proven particularly productive in the mathematics and science education communities (Cobb, Stephan, McClain, & Gravemeijer, 2001; Greeno & The Middle School Mathematics through Applications Project Group, 1998; Lampert & Ball, 1998; Lehrer, Schauble, Carpenter, & Penner, 2000; Roth & Bowen, 1995). However, there is a growing community within educational research that holds that, in order to continue to make progress, the learning sciences need to move beyond extreme, dichotomizing positions that exclusively examine individual knowledge construction or focus only on the ways culture constrains and enables development. Instead, we need to examine the ways in which individual and social processes are mutually constitutive.

The main point of this article is to trace student-learning trajectories to document both the effects that individuals have on a learning community and the effects that the community has on individuals. I demonstrated that individual students' activities, and thereby their mathematical understandings, change as a function of their participation in different social configurations. Furthermore, I investigated the ways in which different classroom social configurations are interrelated and build off of one another. To illustrate these points, I examined how two social configurations within the classroom—local investigations at a computer simulation and whole-class discussions—shape how individual students learn probabilistic reasoning.

The empirical basis for my claims stems from my examination of two case studies that together present a picture of learning within the Probability Inquiry Environment (PIE)—a computer-mediated, inquiry-oriented curriculum designed to help seventh-grade students learn basic probability (Vahey, Enydey, & Gifford, 2000). By looking in detail at student conversations across multiple social configurations, I highlighted how interacting with others influences individual student learning, but in a manner that does not ignore the ways in which an individual's interpretation of events also shapes this interaction. A major implication of my findings is that classroom mathematical practices are developed, in part, for the social or communicative purpose of settling disputes and not purely for their rational or cognitive value. Furthermore, the results reveal the complex relationship between local problem solving and the development of community practices through whole-class discussion. The case studies in this article are important in that they demonstrate how the students can be both actively involved in their own learning and actively involved in shaping the practices of their community.

The organization of this article is as follows. In the first section, I explain the theory and previous studies that ground this work. In the second section, I describe the context for my analysis—a study that my colleagues and I carried out to help middle school students learn probability (Vahey, Enyedy, & Gifford, 2000). In the third section, I outline the methods for my analysis. These methods attempt to synthesize the perspectives of (a) constructivism, with its focus on active learners; (b) sociocultural psychology, with its focus on an active culture and history that shapes individual learning; and (c) conversational analysis, with its focus on the ways in which participants—their minds, cultures, and histories—are made present in the moment. In the fourth section, I present two case studies that together elucidate the role that social configurations played in the learning process. Finally, in the fifth section, I relate the findings back to the sociocultural theories that ground this article and discuss the implications for designing learning environments and improving classroom practice.

THEORETICAL FRAMEWORK: FROM FORMS OF DISCOURSE TO FORMS OF THINKING

At the heart of this article is a straightforward extension of one of the core assumptions of the sociocultural and situated perspectives on cognition and learning—the genetic law of cultural development. Vygotsky (1978) proposed that higher order psychological functions (e.g., probabilistic reasoning) are produced first in social interaction before being internalized by individual students. According to Vygotsky, *internalization* involves the transformation of communicative language into inner speech and semiotically mediated thinking. However, what I mean by internalization here is different than the conventional use of the term. Contemporary interpretations of Vygotsky's writings suggest that internalization does not imply that an external structure has been moved inside one's head (Cazden, 1997; Wertsch & Stone, 1999). Instead, it implies a transformation from socially supported performance to relatively autonomous, competent behavior. That is, higher order psychological functions are initially social in the sense that (a) participation in culturally defined, social activities foreshadow an individual's autonomous competence; and (b) before individuals are able to participate in an activity autonomously, they must first learn to bring their social interactions under self-control through the use of culturally developed sign systems. Although this perspective does not deny that learning involves changes within a person, cognition and social participation are viewed dynamically. It highlights the shift, or transformation of activity, from overt socially supported behavior to more autonomous behavior covertly supported by the adoption of culturally specific, sign systems and forms of discourse.

The zone of proximal development (ZPD), from this perspective, creates the conditions for learning. The ZPD refers to the difference between what a person can do with proper assistance and what the person is capable of alone. However, this difference is not just a gap that defines the leading edge of development, but it also identifies a zone for interaction. The concept of the ZPD defines potential learning and development in terms of a learner's joint activity with other people. By participating in activities that are beyond an individual's current range of competencies, students use the structures of the material and social world to align their activities with a cultural system. By aligning one's individual participation with the ongoing organization of a distributed system that extends beyond the individual's mind, that individual eventually learns how to perform these same functions competently when other aspects of the system are absent (Cazden, 1997; Vygotsky, 1978; Wertsch, 1985; Wertsch & Stone, 1999). The means for development, then, is sustained social interaction and the continual shift toward taking more responsibility for one's own activity.

I did not set out to settle the question of whether or not all higher order psychological functions, or even all types of mathematical reasoning, are first social functions. Instead, I examined a number of examples in which students' competence in probabilistic reasoning can be traced directly back to their interactions with each other, the teacher, and the PIE software. The analyses attempt to extend our understanding of the social origins of cognition by further elaborating on how the social practices of mathematics are developed through interactions that occur across multiple social configurations.

In many cases, what people align themselves with, when they bring their behavior and thinking in line with that of others around them, are cultural practices. I used the term *practice* throughout this article to refer to normatively organized forms of social behavior. Said another way, practices define the legitimate interactional and physical moves within a given culturally defined context. For this article, I emphasized rule-based practices and, in particular, mathematical practices. I used the phrase *mathematical practice* to refer to the shared terminology, speech acts, methods, techniques, and ritualized ways of interacting that a classroom, as a community, has developed to achieve certain mathematical ends. Particular cultural tools provide an infrastructure for these normative ways of reasoning. However, a close examination of the role of these tools is beyond the scope of this article. In this article, I traced how students develop a method for determining the probability of an event. Their method involves first determining all the possible outcomes for a random event (e.g., a coin flip) and then expressing the probability as the number of favorable events divided by the total number of possible events (e.g., the probability of getting a tail is one half). Mathematical practices are critical to understand in that, from the perspective of the curriculum, the generation and adoption of mathematical practices is the explicit objective of the activities.

In defining mathematical practices as a social construct, I did not try to dichotomize the individual and the community. However, a critical difference distinguishes practices from an individual student's locally produced way of reasoning. Practices are descriptions of socially established and taken-as-shared ways of reasoning (Bowers, Cobb, & McClain, 1999)—they refer to the norms of a community. Full participation in a practice implies that one is oriented toward certain aspects of experience, frames one's activity in particular ways, and interacts with the physical and social environment in appropriate ways (Stevens & Hall, 1998). In contrast, an individual student's use or understanding of a practice is what Bowers et al. referred to as a *psychological correlate*—a description of how an individual understands her own and other's social performances. As many scholars have pointed out, the relationship between individual actions and collective practices is reflexive. The ways an individual understands his or her own activity and the meaning of that activity in the context of the community are not separate. First, they are genetically related in the sense that how one understands one's own activity is influenced by how others perceive and react to it. Second, they are interactionally coconstituted. As Saxe (2002) put it, "individual actions are constitutive of collective practices. At the same time the joint activity of the collective gives shape and purpose to individuals' goal directed activity" (p. 277). However, to avoid confusion and ambiguity, I maintained an analytic distinction between the practices and individual's psychological correlates of practices throughout the article.

In this article, I also referred to *discourse practices* as speech acts that are organized by and support students' participation in the larger communities outside of the classroom. Although theoretically distinct, mathematical and discourse practices are not mutually exclusive. Take, for example, a student who argues with her partner that a game of chance is fair because all the possible outcomes are equally divided between two teams. This student is using a specific way of framing the game (i.e., all the possible outcomes) to organize their shared understanding about what is important to consider in the situation. The student's talk must be understood in terms of its relation to her ongoing participation in a classroom where this type of interaction has become commonplace to facilitate the community's joint activity of talking about games of chance. At the same time the student is engaging in an argument. She is making a claim and providing a warrant (Toulmin, 1958). Arguing is an example of a discourse practice that extends beyond the community of the classroom, but it is also recognized within the classroom. These two types of practices, mathematical and discourse practices, are the primary analytical levels at which I examined my data.

It is worth pointing out that norms are not objectively good. They can be aligned with or counterproductive to a given objective. Mathematical and discourse practices that arise in the classroom are no exception. For example, norms can develop that are counterproductive to conceptual learning. One such well-documented norm is the common classroom discourse pattern of initiate–respond–evaluate (IRE; Mehan, 1979). This pattern of discourse fits the definition of a norm and is well adapted to meet certain functions, such as classroom management, within a traditionally organized whole-class discussion. However, it has been shown that the IRE pattern of discourse is not well aligned with the goals of the current mathematics education community, which values a deep conceptual understanding of the material rather than the speed and accuracy of calculations (e.g., Forman, Larreamendy-Joerns, Stein, & Brown, 1998). It is important, therefore, to keep in mind that the value of mathematical and discourse practices analyzed in this article stand in relation to a set of assumptions about what mathematics is and how it is best learned.

One step more general than discourse and mathematical practices are *activity types* or, for simplicity, *activities*. I used the term activity to describe a social event that may include many constituent strips of discourse and practices within it. However, regardless of the number of discourse practices in play, each activity should be a recognizable event or occasion for social interaction. For example, students engaging in making predictions before an experiment was a prototypical activity examined in this article. This activity may be enacted in multiple ways using different combinations of mathematical and discourse practices. In many ways, my use of the term activity is comparable to the notion of a task as it is used in much of the psychological literature. My use of the term activity instead of task, however, is intended to highlight that a task cannot be considered to be located exclusively within an individual.

Activities are both distributed and emergent. They are distributed in the sense that activities are accomplished through the coordinated interactions of multiple participants[1] who may have different orientations toward the activity. The meaning of one participant's actions is determined by the matrix created by their joint interaction and shared understanding of the context. For instance, Hutchins (1995) detailed the ways in which navigating a large ship into a harbor involves the coordinated actions of many participants. The division of labor is such that no one participant is directly responsible for the whole activity, and each participant's actions only make sense in relation to the activities of the others. The same is true for many interactions within a classroom. Activities are emergent in that during ongoing interaction the activity can change or the participant's understanding of the meaning of the activity can change. In this article, activities were different than the typical experimental task in that they were not defined by a prespecified set of actions and constraints that will achieve a goal determined by the experimenter (cf. Newman, Griffin, & Cole, 1989).

A limiting factor in the range and form of activities are *social configurations*—the social and physical settings that the participants use to govern which discourse practices and activities they perceive to be relevant to the situation. The two social configurations explored in this article are (a) students working in pairs at computer simulations (*local configuration*), and (b) the whole-class coming together for a discussion (*public configuration*). These two configurations are important to my analysis because they constrain which types of activities take place and when.

[1]In the most general case, it is possible that a participant is not actually physically present, but is instead represented by physical artifacts or social practices that crystallize his or her participation (Hutchins, 1995).

These different levels of description are used to provide a rich and detailed account of the development of a single mathematical practice as it emerges from activity that is stretched across different social configurations and activities. Similar approaches that coordinate between multiple levels of analysis have been taken by other researchers (e.g., Barab, Hay, Barnett, & Keating, 2000; Hall, 2001). In their study of how students learned about the solar system and the phases of the moon, Barab et al. examined how local group work informed public talk. They outlined two important types of "knowledge diffusion" (p. 744) within and across levels of social organization. First, they outlined cross-group collaboration in which separate, local groups shared resources to complete their modeling activities. Second, they discussed the ways in which knowledge was shared when local groups came together in public settings to discuss and debate their findings. In this latter case, different local models were fused together in the public space.

Another important study that coordinated multiple levels of social organization to understand how different types of discursive practices create different opportunities for teaching and learning is Hall and Rubin's (1998) analysis of Magdalene Lampert's classroom. Hall and Rubin coordinated analyses of private, local, and public settings to trace the development of the concept of the mathematical structure of rate. Adopting the participant's perspective, Hall and Rubin traced two cycles of activity that spanned across the three settings and demonstrated how the students absord the relevant organization of mathematical discourse and content.

Both Barab et al. (2000) and Hall and Rubin (1998) followed the flow of activity across these settings from private, individual activity to local activity to public activity. In this article, I also examined how a mathematical practice emerges from local activity and spreads throughout the classroom via a public discussion. Furthermore, the argument is extended to include a reciprocal analysis of how local activities are transformed by public activity. More generally, I demonstrated that coordinating different levels of description is vital to our understanding of how individuals learn mathematics through participation in a classroom community (cf. Barab & Kirshner, 2001; Hall, 2001; Rogoff, 1995; Vygotsky, 1978).

THE STUDY AND THE PIE SOFTWARE

The PIE was implemented as a 3-week probability curriculum that included computer-simulation games, hands-on games, and whole-class discussions. Each computer simulation was designed to focus on a particular, problematic concept of probability. Note that PIE is a promising environment to explore learning in different social configurations for a number of reasons.

First, probability is arguably among the most used mathematical skills in daily life. Every day people are called on to make decisions based on statistical and probabilistic information. Public opinion polls, advertising claims, medical risks, and weather reports are just a few of the activities that draw on an understanding of probability. In addition, probability is applicable to many academic disciplines such as psychology and engineering. It is routinely used in the professional activities of biologists, geneticists, psychologists, and researchers of almost any discipline. However, many studies show that both children and adults commonly make mistakes when applying probability (Konold, 1989; Metz, 1998; Tversky & Kahnemann, 1982).

Second, probabilistic reasoning can be an authentic activity for young students. Much of students' interest and ideas about how to reason under uncertainty stems from playing games. Games and game playing are an ubiquitous part of childhood in most American cultures, and in many of the games children play the outcome of the game is based on chance—on the outcome of random devices such as dice, cards, or spinners. The PIE, then, builds on the interests of young students to help them develop probabilistic reasoning skills that will be invaluable to them in their professional and daily activities as adults.

Perhaps more important for this argument, the PIE software was designed to promote specific interactions in the classroom culture. As designers, Vahey et al. (2000) framed the PIE as a context for collaboration. We attempted to design the software to spark and support productive conversations between students and between students and their teacher. For example, we wished to make agreements and disagreements visible (Bell & Linn, 2000) and solvable within the interactional space afforded by the interface. One way we attempted to operationalize making disagreements visible was to create one space where students had to construct a shared response, but gave each student "agreement bars" to record the extent to which the shared answer reflected their own thinking. This sometimes led to extended negotiation when one student pulled the agreement bar down to show she disagreed with the prediction that her partner constructed without her input. To make disagreements productive and solvable within the environment, we structured students' predictions to have both a claim and a warrant (Toulmin, 1958). We also provided a large set of representational resources that served as a ground and data for their arguments. For a more detailed description of the environment in relation to our pedagogical objectives and learning philosophy, see Vahey et al. (2000).

In the PIE, students actively investigate probability by trying to figure out if particular games of chance are fair to all participants. The students' collaborative activity is structured around: articulating their intuitions, systematically testing their ideas by gathering and analyzing empirical data, and communicating their revised understanding of the domain to their classmates. This combination of active, local investigation and public presentation makes the PIE an interesting environment within which to explore student dialogue and the interplay between social configurations.

In the computer games, student investigations into the fairness of specific games of chance were structured such that the student was guided through a cycle of inquiry (cf. White, 1993). Each PIE activity consisted of six steps: Rules, Try, Predict, Play, Conclude, and Principles. Each one of these steps serves as an identifiable stage that supports certain kinds of interaction.

In *Rules,* the software shows the students an animated introduction to the current game. In *Try,* the students get a chance to experiment with the representations and controls of the simulation. This was done to allow the students some amount of familiarity with the environment before asking them to make predictions. In *Predict,* we chose questions that highlight aspects of the game that are particularly salient to a normative understanding of probability. In *Play* (Figure 1), the software simulates different games of chance. At this stage, the PIE provides several resources and capabilities to facilitate productive collaboration including coins, probability trees, bar charts, and frequency tables (Enyedy, Vahey, & Gifford, 1997). In *Conclude,* the students compare their predictions to the data from the simulation. Finally, in *Principles,* the environment scaffolds the students to jointly articulate what they can generalize from their activity.

These simulations are then followed by hands-on games in which students flip coins, roll dice, and so on as they investigate aspects of probability without using the computer environment. Throughout the curriculum, the students also participate in whole-class discussions in which each pair reports their findings, compares their findings to that of other groups, and discusses the general mathematical practices that can be derived from this game and applied to the analysis of subsequent games. The computer simulations, hands-on activities, and whole-class discussions each lasted approximately 40–50 min. Over the 3 weeks, the students spent approximately 8–10 hr of focussed activity engaged with the curriculum and each

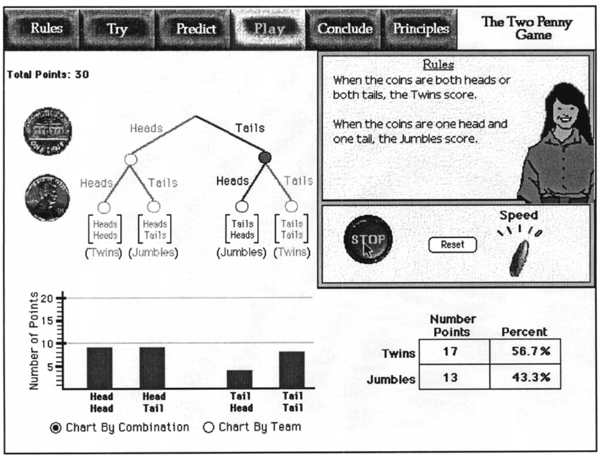

FIGURE 1 The PIE, shown here in Play mode of the Two-Penny Game.

other around issues of probability (for a more thorough description of the environment, see Vahey et al., 2000).

Setting: Students, School, and Teacher

The PIE curriculum was implemented in two 7th-grade mathematics classes in an urban middle school. The same teacher taught both ethnically diverse classes: 42% White, 29% African American, 13% Asian, 9% Hispanic, and 7% other. In additionally, many of the students were from lower socioeconomic status households. Thirty eight percent of the students in the school received free or reduced-price lunches.

The middle school mathematics classrooms of this school did not have computers. Instead, on days when computer activities were scheduled, the class visited the school's computer lab. The computer lab consisted of about 20 older model computers. The computers were positioned around the perimeter of the room. In the center of the room were some additional tables. On days when the students did not use the PIE software, the class was held in the regular classroom.

METHODS

Two classes using the PIE software were extensively videotaped. The two researchers who made these recordings (a colleague and myself) were nonparticipant observers (Becker & Geer, 1969). We were present during all sessions of the PIE curriculum, but we provided no assistance with the classroom instruction. However, we were participants in the classroom in a less formal sense of the term. We were there every day, and although we rarely directed questions to the students, we did answer questions directed to us. It was known throughout the class that the two researchers were also responsible for the design of the PIE curriculum and computer games. One result of this was that the teacher would occasionally consult one of us in front of the class on some issue (e.g., how the random generator of the computer worked). In addition, we were often asked to provide technical help on the computers. Therefore, although we did not participate in the cognitive or content-related issues of the class, we did have a relationship and a role within the class that should be kept in mind when reviewing the transcripts of the video recordings.

Data Sources

Video data was collected from local (e.g., pairs working at a computer) and global (e.g., whole-class discussion) perspectives. For all of the PIE games and hands-on activities (except for the whole-class discussions) the students worked in pairs. We followed four focal student pairs, two pairs from both PIE classrooms, each class period throughout the entire curriculum. The four focal student pairs, referred to throughout this article by pseudonyms, were nominated by the teacher to represent a range of gender, ethnicity, and ability levels. These pairings remained the same throughout the 3 weeks, except in cases of absences or what the teacher considered disruptive behavior. The degree to which the focal pairs represent a wide range of gender, ethnicity, and ability can be seen in Table 1. An additional camera rotated though the nonfocal pairs in each classroom recording each pair of students during their work for 5–15 min at a time. This videotaping strategy provided both detailed case studies and a sampling of the rest of the classroom. For this article, however, I analyzed the interactions of only two of the four possible focal pairs.

These two case studies were not chosen for analysis based on the students' performance on the posttest. Instead, the cases were chosen because the students engaged with the software and with each other in a way that made their learning process available for analysis. First, the students in these cases adopted the instructional objective of the activity in that they were actively committed to figuring out if the games were fair or not. Second, they were engaged with each other in the process. Furthermore, as shown in Table 1, their performance on standardized tests and our preassessments put them reasonably close to the mean (with the exception of Rosa and Mike). One student, Mike, scored very poorly on standardized tests administered before the intervention. In additional, although Mike scored at the mean on the pretest, he was rated as a low-achieving student by the teacher. Three of the six students that were involved in these two case studies, Will, Derek, and Maria, had standardized test scores close to the mean, pretest scores at the mean, and were rated by the teacher as medium-level mathematics students. Two

TABLE 1
The Four Focal Pairs That Were Videotaped

Focal Group	*Name*	*Gender*	*Ethnicity*	*Standardized Test Score*[a]	*Pretest (Out of 20)*	*Posttest (Out of 20)*
1[b]	Mike[c]	Male	African American	14	8	6
1	Robert	Male	African American	55	5	9
2[b]	Derek	Male	African American	51	9	13
2[b]	Will	Male	African American	61	8	11
3	Jo	Female	Latina	51	5	11
3	Sandy	Female	African American	63	6	7
4[b]	Maria	Female	Latina	64	10	12
4[b]	Rosa	Female	Latina	97	14	15
Class average	62	10	14			

[a]The score reported is based on the average of reading, comprehension, writing, math, and problem solving scores on theIndividual Test of Academic Skills from School Research and Science Corporation. [b]These mark the students analyzed in case studies described in this article. [c]Mike, but not his partner, is shown here because he partnered with Rosa for the Three-Coin game in Excerpt 6.

students in Table 1, John[2] and Rosa, were arguably better students. They scored 1 standard deviation above the norm on both the pretest and standardized test scores, and they were rated as high performers by the teacher. However, during the pretest and during the activities all six students expressed many of the same naïve and limited intuitions about probability (for examples of common intuitions, see Konold, 1989; Konold, Pollatsek, Well, Lohmeier, & Lipson 1993; Vahey et al., 2000).

However, it is important to be clear that the unit of analysis for these cases is not simply the individual students. Instead, I focused on the learning trajectories of the students. These trajectories often span across multiple combinations of actors, activities, and settings. By using learning trajectories as my unit of analysis, rather than individual students, I attempted to restore and examine ways the context and social interaction are involved in a single student's conceptual change, without ignoring the individual's contribution to creating the context and interaction (cf. Barab, Hay, & Yamagata-Lynch, 2001; Cobb et al., 2001). Thus, my choice of analyzing learning trajectories is consistent with a, "commitment to agent-in-settings as unit of analysis, and to the contention that cognition occurs and is given meaning through the dynamic relations among the knower, the known, and the evolving context which knowing occurs" (Barab & Kirshner, 2001, p. 9).

During the local investigations, the video cameras were arranged in such a way that the computer screen (or work surface in the case of hands-on activities), faces, and gestures of the students were visible on the tape recordings. During the whole-class discussions, a camera was set up at the side of the room with microphones in both the front and back of the room to record clearly all the dialogue of the classroom.

A database integrated in the PIE software captured all student actions within the PIE. The database recorded every pair's predictions, observation notes, conclusions, principles, agreement bar entries, and navigation routes through the different modes and screens of the environment. The main use of the students' work at the computer was not to assess student understanding during the transitional points along the students' learning trajectory. Only the thinnest traces of the students' reasoning and competencies that were displayed in their dialog and interaction with each other were actually inscribed into the database. Rather, the database was invaluable as a complement to the video recordings in that it helped to recreate the representational state for any given moment that we wished to examine in the video record. In other words, the database in this study was used primarily to help recreate the material circumstances of any episode of interaction that we qualitatively analyzed in detail.

I used a set of overlapping research methods for my analysis of the video data. First, I identified segments that were good candidates for a close analysis of interaction (Erickson, 1992, 1998; Erickson & Shultz, 1981; Goodwin & Heritage, 1990;

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[2]John is not shown in Table 1 because he was not a focal student. He is mentioned, however, because he played a pivotal role in the whole-class discussion.

Jordan & Henderson, 1995). These excerpts were chosen because they fell into theoretically meaningful categories and because I considered them to be critical interactions along the students' learning trajectories. For the most part, they consist of when students are making predictions, considering new information, reflecting on or reconsidering their ideas, drawing new inferences, or arguing with their peers

After choosing the excerpts, I examined the interactions to produce my initial coding scheme or analytic categories that elucidated how students were learning mathematics in this particular case (Charmaz, 1983; Erickson, 1992, 1998; Erickson & Shultz, 1978/1997; Glaser & Strauss, 1967; Hall, 2001). This analysis provides a way to illuminate critical moments along a learning trajectory and the important analytical dimensions of these moments. What is presented is the minimum set of interactions that preserves the critical moments for these particular students' learning trajectories. Many noteworthy interactions and analyses have been omitted for the purposes of this discussion.

Obviously, this data reduction is a necessary part of research and concurrently introduces a subjective choice on the part of the analyst that should be open to scrutiny. The trustworthiness and reliability of the decisions and choices made in this article came from three aspects of my methods. First, I relied on a network of interconnected data sources that are triangulated and coordinated to support my conclusions. These sources include the research literature, student scratch materials, worksheets, predictions, and conclusions inscribed into the PIE software, and the students' pre- and posttests. Initial conjectures are tested against these multiple data sources for confirming and contradictory interpretations. Second, other researchers vet the interpretations of the case in informal and formal data analysis sessions. These sessions often either produce additional evidence for an interpretation or provide alternative interpretations that are then tested against the record. Third, interpretations of activity are tested by looking "downstream" at the student's subsequent interactions to see if the interpretation is consistent with the student's own subsequent actions and talk.

I caution readers about the representativeness of these cases. I did not intend to suggest that these two trajectories represent the only possible trajectories for learning probability or even that these two account for all the pathways within these two classrooms. It is clear that issues of generalizability need to be addressed using further case studies as well as other methods.

Instead, the power of these cases resides in the ways they identify and demonstrate some of the important interrelations between individuals and their participation in different types of practices and configurations within a learning community. It is within this trajectory of participation—distributed over time, space, and multiple social configurations—where we begin to be able to see the ways in which individuals construct disciplinary knowledge, the ways in which a community shapes and leads that knowledge construction, and the ways in which individuals contribute and shape their community.

ANALYSES

The primary learning objective for the part of the curriculum that sets the stage for these cases is a conceptual understanding of the outcome space and its value in reasoning probabilistically. The outcome space is a classic way of reasoning about probability based on considering all the possible outcomes of an event, partitioning these outcomes into favorable and unfavorable events, and quantifying the relationship between favorable outcomes and the total number of possible outcomes as a fraction. For example, there are four possible outcomes (i.e., the outcome space) of two coin flips: heads–heads, heads–tails, tails–heads, and tails–tails. If you wished to predict the probability of an event, such as the two coins landing with the same side face up, you first might partition the four outcomes into favorable and unfavorable events (e.g., heads–heads and tails–tails vs. heads–tails and tails–heads). You could then quantify the favorable events and express the probability as a fraction—as the number of favorable outcomes over the total number of possible outcomes (i.e., two fourths). This way of reasoning is one of the primary methods endorsed by the mathematics community (National Council of Teachers of Mathematics [NCTM], 2000).

However, establishing that the PIE was a successful way to help students learn probability is not the primary concern of this article. The analysis here is concerned with how students learn in this context. What aspects of the software contributed to student learning? And how did the manner in which the software was integrated into the classroom community mediate these learning outcomes?

To answer these questions, I followed two pairs of students as they learned how to use the outcome space to reason about probability. In particular, I used these cases to examine the way that interactions are distributed over time and across physical and social configurations. In the first case study, the interactions were stretched across two radically different social configurations—local investigations of games of chance at the computer and whole-class discussions. Interactions across these two social configurations dramatically show the ways in which the social configuration can shape interactions and thereby influence student learning. Previous work in this area has noted ways that physical[3] and social configurations restrict participation, but it has not explicitly demonstrated the connection between participation and learning (e.g., Roth, McGinn, Woszczyna, & Boutonné, 1999). In the second case study, I closely examined the local context to illuminate the ways that discourse and mathematical practices interact during student learning.

[3]Although it is clear that the physical and symbolic ecology of the PIE also plays a crucial role in shaping these learning trajectories and the students' interactions, a close examination of the role of individual representations is beyond the scope of this article.

Case Study 1, Rosa and Maria: From *Models of* to *Models for*

In this case, the students developed empirical models of the game and empirical methods to produce probabilistic inferences using computer activities. However, in the whole-class discussions, the students (with the teacher's help) developed generalized models for reasoning and making probabilistic inferences based on the outcome space. These interactions show that the two social configurations play very different roles in student learning. Specifically, students' locally produced ways of reasoning were transformed into mathematical practices in the public arena of the whole-class discussions.

A critical feature of mathematical practices is that they are prospective. Practices address generalized methods that will be applicable to many future situations. In contrast, most of the ways students reasoned while completing their local investigations were reflective, descriptive models of their activity, of the game, or of the data they produced. That is, at the computer the students were focused on producing retrospective descriptions of things that had already happened. Mathematical practices, conversely, are models for reasoning and arguing about specific mathematical ideas that will be generally applicable to future situations (Bowers et al., 1999). As Cobb (2002) pointed out, mathematical practices integrate at least three levels of community norms. First, mathematical practices are defined by a community's joint enterprise—a normative purpose. Second, they involve recognized patterned ways of interacting—a normative participation structure. In Cobb's empirical work he often concentrated on normative standards for argumentation. Third, mathematical practices involve particular ways of reasoning and using tools—a normative set of rules and tools to solve reoccurring problems. Although it is true that every prospective, generalized practice must be instantiated in a specific situation, the difference between a model of one's activity and a prospective mathematical practice is that mathematical practices are designed to make disparate situations similar.

Mathematical practices can be seen as producing sameness across a variety of situations that are similar in one or more dimensions (Lobaoto, 1997). In contrast, local *models of* a situation can be seen as a way of understanding each situation as a unique event. From this view, *models for* are seen to play a central role in generalization and transfer. The process of mathematical generalization has to do with constructing continuity by construing different situations to be the same. Learning what exactly about a situation generalizes is often arranged by social and cultural contexts (Jurow, 2002). In particular adults often engineer transfer by pointing out similarities across contexts, sequencing activities in particular ways, and pointing out when prior knowledge is relevant (Brown, 1989; Lobaoto, 1997). These social interactions link contexts together to allow for comparisons to be made and conjectures of sameness to be constructed (Jurow, 2002). Eventually, these social supports are fused with the material and symbolic tools to create representational practices that are applied across contexts as generalized models for reasoning.

This shift from individual ways of reasoning as *models of* activity to mathematical practices *for reasoning* is found in the PIE data set. The two main types of *models of* activity that the students engaged in were modeling how the game worked and the data produced by the simulation. The *model for* reasoning that the class eventually adopted is the use of the outcome space to determine the fairness of games of chance. In the series of interactions that follow, I traced how students' local efforts to make sense of the PIE games are transformed into the normative practice for the classroom community.

Day 1: Models of

A prototypical example of a *model of* data can be found in an interaction in which a pair of students interprets the data that the PIE simulation has produced during their investigation of an activity titled the Two-Penny Game. In this game two teams, the Twins and the Jumbles, compete for points. If two coin flips land on the same side, the Twins score. If the coins land on opposite sides, the Jumbles score. The game is, therefore, mathematically fair—two of the four equally likely outcomes are assigned to each team. However, in their predictions the two girls in this case study, Rosa and Maria, predicted that the game will be unfair because the Jumbles have the outcomes the girls believe to be more likely.

Immediately prior to when I began my analysis of the first case (Excerpt 1), the students are asked how many times each outcome will happen. They develop the idea that mixed outcomes (e.g., heads–tails and tails–heads) will be more likely. This intuition is quite common in students of this age, and it is related to what others have called a heuristic way of reasoning based on what they consider to be a representative outcome for a random event (Tversky & Kahneman, 1982). Having students confront and elaborate this intuitive way of reasoning was one of the primary reasons we designed this game (Vahey, Enyedy, & Gifford, 1997; Vahey et al., 2000).

Excerpt 1: Rosa and Maria predict that the Jumbles will win.[4]

1. Rosa: Jumbles I think is gonna win…whoa I said they wouldn't be exactly the same, right?
2. Maria: Uh, yeah, no, um, could make it higher.
3. Maria: [manipulates the bar graph to show the Jumbles will score more than the Twins by a ratio of 6 to 4].

[4]The transcription conventions used are adapted from Gumperz (1982). Brackets are used to identify comments by the research, equal signs are used to identify where student speech overlaps, and ellipses are used to identify a pause.

In Excerpt 1, Rosa's prediction that heads–tails and tails–heads are more likely leads the two girls to logically conclude that the Jumbles, who score when these more likely outcomes occur, are likely to score more points. As part of their prediction they constructed a bar graph of their quantitative expectations (see Figure 2).

These predictions are important in that they create a reason for the students to engage in the data modeling that will go on during the simulation. They have created an object, the bar graph, to compare with the actual results of the game. Furthermore, they have a stake in that outcome.

Figure 3 and Excerpt 2 are examples of these students modeling the data from their simulation. In the first part of the interaction, they are rooting and cheering for the team they predicted will win (i.e., the Jumbles) and relating their interpretation of the data to their expectations.

Excerpt 2: Rosa and Maria rooting for the Jumbles and constructing models of the data.

1.	Rosa:	(Gasp) It's getting close! 'Cause the Jumbles are still ahead.
2.	*Computer:*	*At 200 points the Jumbles are ahead. To keep playing, press the start button.*
3.	Maria:	The jumbles==[starts a new game]…Man! Come on. Man! Come on.
4.	*Computer:*	At 20 points the Twins are ahead.
5.	Rosa:	Who cares about the Twins? We don't like the Twins!
6.	*Computer:*	*At 200 points the Twins are ahead.*
7.	Rosa:	By like two points! Three points.
8.	*Computer:*	*Do you think the game is fair or unfair now?*
9.	Maria:	I think it's fair.

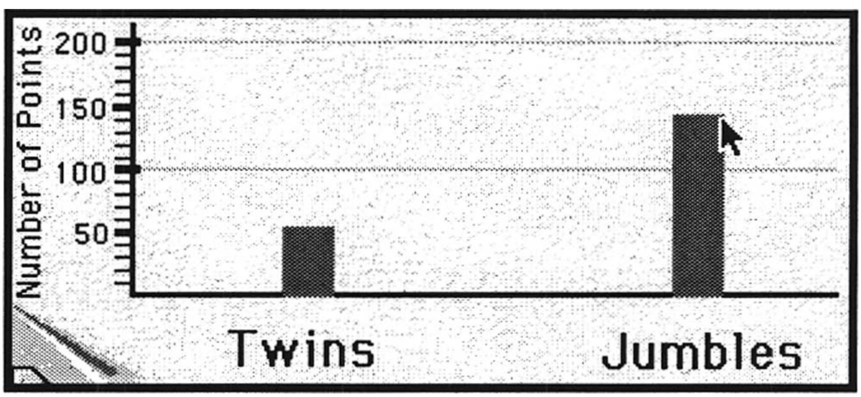

FIGURE 2 Their predicted ratio of points.

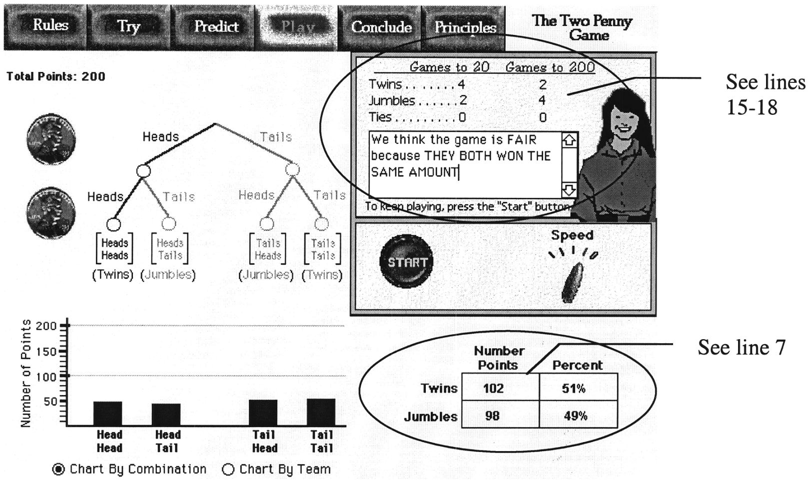

FIGURE 3 State of the game for Rosa and Maria's observation question.

10. Rosa: The game is==
11. *Computer: ==To play, press the start button.*
12. Rosa: =='Cause they won like, equal, you know.
13. Maria: It's fair
14. Rosa: It's fair.
15. Maria: Because…they won an equal amount.
16. Rosa: They've what? Won an equal amount, right?
17. Maria: Yeah.
18. Rosa: [Typed: We think the game is FAIR because, THEY HAVE WON AN EQUAL AMOUNT.]

In turn 1, Rosa is calling attention to and interpreting the bar graph display as she roots for the Jumbles to win. Rooting for one team was both a common and important activity during the simulation. In this case, Rosa makes an observation about the progress of the game by interpreting an important representation and she relates the information back to their expectations for the game. Rooting for one team to win encourages the students to notice and interpret representations that provide new insight about the phenomena that is relevant to their activity.

In turn 4, the computer stops their simulation to inform them who is ahead after 20 turns. Rosa, however, contests the computer's interpretation of the data and animates the chart of points per team to produce an alternate assertion (turn 7). This argument with the computer is extended in turn 12. In this part of the exchange, Rosa references the chart that displays the number of games each team has won.

Their reaction to the computer's statement prompts the students to collaborate on an interpretation of the data and their understanding of the game. This *model of* the game is based on a particular interpretation of the data that diminishes the importance of the differences between the two team's points and number of games won. In generating this descriptive *model of* the data, the two girls change from a nonnormative expectation about the game (i.e., that the Jumbles will win) based on a nonnormative explanation (i.e., because mixed outcomes are more random and, hence, more likely) to a normative expectation (i.e., that the game is fair) based on an empirical explanation (i.e., that the teams have won the same number of games and scored about the same number of points). This represents a significant step forward along their learning trajectory. They have begun to attend to one of the major resources, the data, that defines probability as a normative practice (cf. Stevens & Hall, 1998). However, they have not yet demonstrated the normative method of probabilistic reasoning from the point of view of the curriculum. Their current method still requires the students to generate data empirically and make their inferences based on that data. The two girls have not yet adopted a method of reasoning that is generative and that will allow them to reason about new games without playing them first. In other words, they have not yet developed a *model for* reasoning that will allow them to make a number of future games the same.

Day 2: Whole-Class Discussion

Consensus building. The whole-class, public social configuration provided a critical set of interactions that contributed directly to Rosa and Maria's learning trajectory. However, the analysis of these interactions necessitates a switch from examining the development of individuals through social participation to examining the development of the community itself. The whole-class discussion can be divided into three consecutive activities. First, each pair of students is given the opportunity to report their results. Second, as a class they attempt to reach a consensus about what was important about this investigation and what the results mean. Third, the teacher attempts to maneuver the students toward thinking about the phenomena prospectively and to invent a mathematical practice.

In the following discussion, which occurs the next day, they report a number of locally consistent accounts of what happened when creating a larger set of pooled data from which every student can reason. In Excerpt 3, Rosa reports her local version of the simulation.

Excerpt 3: Rosa reports their results.

1. Teacher:Did the Twins win any of them?
2. Rosa: They won, well actually they pretty much tied when the games were at 200 flips. … But, at twenty flips, the Jumbles were always ahead.

3. Teacher: So, at twenty the Jumbles were ahead but by the end, it was ==
4. Rosa: ==almost even.
5. Teacher: Almost even.
6. Rosa: The Jumbles won seven games the twins won six.

However, many of the other pairs of students had drawn different conclusions from their experiences with the simulation. Some students found that the Twins had won more games, and others found that the Jumbles had won more games. These conflicting findings juxtaposed in the public setting make the issue of the game's fairness—which for most students had been settled in the local social configuration—problematic once again. Natasha makes this problem explicit in Excerpt 4. Natasha then reframes her local data in the larger trends of the class' pooled data.

Excerpt 4: Natasha reasons from the pooled class data rather than just her own data.

1. Teacher: Natasha, do you agree? Fair or unfair?
2. Natasha: Um, I think it was sort of fair even though the Twins won a lot of the games.
3. Teacher: Why do you think it was fair if the Twins won more?
4. Natasha: Well, because um, hearing about other people, they got Jumbles more, and we got Twins more, so I think it's about the same.

John and the whole class: Models for. More important than consensus building, however, is the qualitative shift in the type of reasoning practices that are developed exclusively in the whole-class discussion. In the next example, I examined the negotiation of a mathematical practice that provides a generative and prospective method to make a normative inference about probabilistic situations like the PIE games. In the following excerpt, negotiating a practice—like modeling the data in the case of Rosa and Maria—still relies on collaboration to produce an interactional explanation. However, it differs in that the focus of the exchange is on developing a prospective model for reasoning about the fairness of games that can be applied to future situations.

Although Rosa and Maria are not active participants in Excerpt 5, it is nonetheless an important part of their learning trajectory and is, therefore, included in the case study. Even though they do not contribute to the exchange, they appear to have listened closely. As we saw, immediately prior to this discussion the girls reason about the game's fairness based on the empirical data they have collected (i.e., the number of points scored and the number of games won). Immediately following the discussion, the girls reason about games of chance in a qualitatively different way. They use the abstraction of the outcome space to organize their reasoning.

Excerpt 5: John and the teacher publicly construct a practice that involves quantifying the outcome space.

1. Teacher: Okay, um…let me ask you this. What if I just showed you this set up. And you didn't even play the game yet. I mean how many of you predicted that given this and given sort of the ex…stop that…given the explanation that Derek gave, which was a good explanation of how the game worked, how many of you predicted that it would be a fair game? John, would you say fair game to start out?

[teacher puts up a screen shot from the game with a probability tree]

2. John: Yeah.
3. Teacher: Why? If you just look at the set up of it.
4. John: Because they each have an equal chance of winning. There's four ways that the coins can land. And the Jumbles can win two of them and the Twins can win two of them.
5. Teacher: What do you mean four ways for them to land?
6. John: There's heads tails, tails heads, tails tails, and heads heads.
7. Teacher: So these four here? [points to the probability tree] And what did you say about two of them?
8. John: The jumbles can win two heads tails, tails heads, and the twins can win two which are heads heads, and tails tails.

Excerpt 5 takes place during a whole-class discussion following the Two-Coin Game. Even though this interaction only involves the teacher and one student, John, the conversation has implications for the class as a learning community as well as Rosa's future investigations. For both Rosa and the rest of the class, this is the first public production of a way of reasoning that will eventually become the community's norm.

In Excerpt 5, the teacher changes the nature of the activity by introducing a hypothetical situation in which the students are not allowed to produce empirical data to induce that the game is fair. In response, John invents a practice that involves comparing the quantity of outcomes for each team.[5] The instructional move that leads to this interaction was largely an insight of the teacher. We provided the teacher with a number of overheads that addressed the outcome space and suggested that he encourage the students to discuss the relevance of the outcome space. However, we did not provide any sort of script that addressed how to work this topic into the conversation.

[5]Note, however, that John was not one of our focal students, and so we cannot be sure if this way of reasoning was invented in this interaction or prior to it. Still, it is clear that this was the first time that a large number of students, including Rosa and Maria, were exposed to this way of reasoning.

In this example, the shift from descriptive *models of* data to prospective *models for* reasoning was initiated by the teacher's involvement. By proposing a "what-if" scenario, the teacher changes the interactional context in two important ways. First, the teacher limits the set of representations from which the students can draw by limiting them to using the set up of the game represented by a probability tree placed on an overhead projector. Second, the teacher limits the set of permitted actions that the students could perform within the situation by cutting off any empirical investigations to determine the probabilities. This second limitation is critical because it facilitates the move from modeling empirical data—specific to a particular situation—to a prospective practice that can be applied to a wide array of situations. In these two ways the teacher is modeling the use of the probability tree as a tool for concretizing the abstract outcome space and simultaneously pushing the students to consider the value of constructing a *model for* reasoning.

Within the class, John's method is the first time anyone has publicly counted outcomes to determine if the game is fair. In doing so, he created a new function for a material tool that had up to that point been used by the students mainly as a game board to keep track of which team scored (Enyedy, Vahey, & Gifford, 1998). John's model for determining fairness emerges from a combination of factors. Saxe's (1991, 2002) emergent goals is a useful framework to help explain the emergence of this important shift. First, it emerges as a new goal that is taken up by John through his interaction with the teacher (i.e., to determine the fairness of the game without data). Second, the details of the model itself are generated interactively with the teacher over seven conversational turns. Third, certain participation structures and discourse practices, such as providing warrants and backings for claims, shape their interaction. Fourth, the tree provides a material anchor for the conversation and actions that generate a solution. Fifth, John's and the class' prior history contribute important constraints and checks on the model. In this case, because the class had already decided collectively that the game was fair based on the data, the new method also had to lead to the same conclusion. The combination of these factors helps explain the infrastructure that makes John's creative leap forward possible.

For the development of the community's mathematical practices, and for individual students who will come to adopt this method, this interaction marks a significant change in the available ways of reasoning about probabilistic situations. For Rosa, at least, this seems to hold true. The next day, she used this newly established mathematical practice to reason about a game she is encountering for the first time. A possible explanation is that her passive participation in the discussion, combined with her active participation with the software and her partner, was sufficient to initiate this conceptual change. If true, this is a perfect illustration of Vygotsky's genetic law of cultural development in action. Rosa's participation with others, and in particular with John as more competent other, leads to the genesis of Rosa's conceptual change. This change in reasoning is first seen in social interaction, in her interaction with Mike, and later is seen in her solitary competence on the posttest.

Day 3: Local Interactions as a Mechanism for Propagation

John's public explanation, however, was not enough to align everyone's reasoning with this new practice immediately. Many of the students needed further opportunities to understand the relevance of the outcome space. Other studies have demonstrated the value of opportunities to apply and practice what is learned through collaboration, but these studies have only examined the cases wherein individuals, in isolation, practice the skills they have learned collaboratively (Webb, 1989, 1994). In the next example, I show how collaboration can be valuable to more advanced students as a context for practicing a learned skill by teaching it and simultaneously valuable for the less advanced student as a context for learning.

Cycling back to the local social configuration also played a critical role in the propagation of the practice throughout the class. When students began their local investigations of the next game, arguments between students became opportunities for one student to discipline another in the use of the practice of counting the outcomes in the outcome space. Students' local investigations were transformed by the students' shared history. Before, collaboration in the local investigations served to explicate a model of the game. Now, student collaboration at the computer was an opportunity to establish and practice the community's shared ways of reasoning. A clear example of this sort of teaching and learning moment (cf. Hall & Rubin, 1998) in which one student models for another how to use the practice in this new situation is shown in Excerpt 6.

In Excerpt 6, Rosa—now paired with a new partner, Mike—uses a disagreement about the game's fairness as an opportunity to teach him the classroom practice of quantifying the outcome space. In the Three-Coin Game, two teams—team A and team B—compete for points. The outcome of three consecutive coin flips determines which team scores a point. Team A scores a point on five of the eight possible outcomes. Team B scores on the remaining three outcomes (see Figure 4). This game is mathematically unfair because team A scores on more outcomes than team B. The Three-Coin Game was designed to highlight the importance of the partitioned outcome space in determining the probability of an event or judging the fairness of a game of chance.

Excerpt 6: Rosa teaches Mike the classroom practice of counting the outcome space.

1. Mike: Oh! Okay, do you think the game is fair. We think the game is fair. Go to, um…[points to the radio button for "Fair Game"]
2. Rosa:s No we don't!
3. Mike: We don't think it's fair?
4. Rosa: No! Because lookit, 1, 2, 3, 4, 5. [points to each outcome as she counts] They get five chances and B only gets three chances.

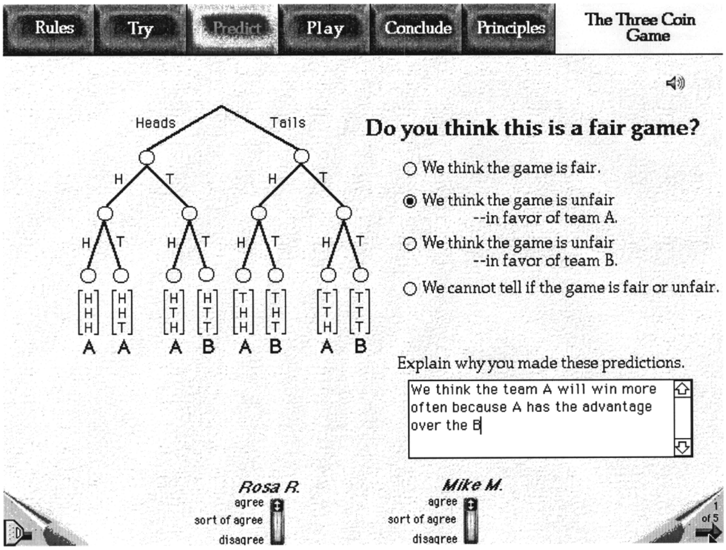

FIGURE 4 The reproduction of the counting practice.

5. Mike: All right, um, unfair, unfair. [recounts outcomes]
6. Rosa: In favor of team B, I mean team A. Right?
7. Mike: We think this team is unfair in favor of team A, okay. We think… Um, that A will win more often because…because…
8. Mike: [Typed: We think the team A will win more often because A has the anveng of the B] [He meant to type advantage.]

In this short interaction, Rosa claims the game is unfair because team A has more chances. In her public reproduction[6] of her inference, she seems to equate fairness with each team owning an equal number of outcomes. Her procedure for comparing the teams' outcomes involves three steps. First, by counting off team A's outcomes in turn 4, she implicitly parses the outcomes into two classes, team A and team B. Second, with the same utterance she compares the quantities for these two classes. In turn 4, Rosa associates each team with a single number corresponding to their total number of outcomes, "they get five chances and B only gets three chances." This utterance reduces the complexity of the situation by eliminating the relevance of which particular outcomes belong to which team. Third, in turn 6, she provides the outcome of her comparison of the two quantities. It is significant that

[6]In turn 2 she says "no," but it is not until turn 4 that she begins her public performance of her reasoning.

this turn is a correction of Mike's reproduction of her reasoning. In turn 6, Rosa elaborates on Mike's turn and demonstrates a more complete conclusion. She implies that it is important to know more than that the two partitions are unequal, but that you also need to know in which direction the inequality lies. With the method for reasoning established publicly, Mike is able to continue the interaction and make the appropriate inference. Furthermore, he produces a written justification that is consistent with that inference, that team A will win more often because they have more outcomes.

This case study has shown the way participation was transformed as it moved across different social configurations and how these transformations contributed to both individual conceptual change and the development of community practices. Each configuration for interaction (a) provided unique contributions to these individual and collective processes of development, and (b) built on the shared history of local pairs of students and the class as a learning community.

Case 2, Derek and Will: A Detailed Account of Social Performance Before Competence

The purpose of this second case is not to argue for the generalizability of these findings but to further explicate the importance of the interplay between different social configurations and expand on the ways in which one discourse practice contributes to that interplay. For this analysis, I focus on an argument that occurred between two students, Derek and Will, as they engaged in the Three-Coin Game (described in the previous case and Figure 4). They stretch their argument across two different activities—making predictions and interpreting data—within a single social configuration—their local investigation of the game. Their argument takes on different characteristics in the two activities, and both parts of the argument advance their learning in different ways. First, by collaboratively making a public prediction, they create an intersubjective disparity—a difference of opinion that organizes their subsequent interaction. Second, in the context of the data and displays generated by the computer simulation, they are able to coordinate a number of resources to converge on a common understanding of the game and resolve (to some degree) their differences in understanding.

This series of interactions establishes an example of competence that can be traced back to social interaction, in this case social conflict. I show how one boy's reasoning, Derek's, is reorganized by participating in this argument with his partner. Unlike Piaget's (1932, 1983, 1985) account of peer interaction leading to cognitive conflict, it is not Derek's reflection on the differences between Will and his own opinion that leads to this reorganization as much as it is the social pressure to resolve their disagreement. It is the public nature of the disagreement that calls out for the production of justifications and evidence (in fact, as I show next, it is his partner, Will, who literally calls out for a justification). Furthermore, it is in social interaction where these justifications and other productive learning behaviors are produced and elaborated.

In the first prediction question of this game, Will and Derek articulate a jointly constructed prediction that the game is unfair.

In turn 1 of Excerpt 7, Derek explicitly states that the game is unfair. In turn 2, Will silently, but visibly, counts up the outcomes along the bottom row of the probability tree with his finger and apparently elaborates on Derek's statement by adding that team A will win more often. In turns 3–7, both students jointly construct the statement that team A has more opportunity. Their prediction, on its surface, is entirely consistent with the normative view that team A has more outcomes (or, as Will and Derek said, "slots" or "opportunity").

Excerpt 7: Derek and Will predict the game is unfair because team A has more opportunity.

1. Derek: We think the game is unfair. ==We think that the game—team A will win more often because==
2. Will: [talking at the same time at the same pace as Derek] ==We think that team A—…will win more often because== [points to team A's outcomes as he counts along the bottom row]
3. Derek: ==It, I mean they=
4. Will: =have more…have more
5. Derek: Have…more=
6. Will: =more slots.
7. Derek: Opportunity.
8. Derek: [Typed: We think the team A will win more often because they have more opportunity.]

A Disagreement Becomes Visible

The third prediction (I have omitted the second prediction question because it is not relevant to the current analysis) question asks the students to show how often each team will score by dragging the bars on a bar chart to the height they expect them to reach during the game. Their response to this question is a turning point in this activity because it is the first time that the two students explicitly discuss their different expectations for the simulation.

In Excerpt 8, Derek immediately drags team B's bar higher than team A's bar, which conflicts with what Will expected their prediction to be. Derek also states he thinks that Team B will win (turn 3). Derek's justification for this conclusion, however, is complicated and requires some additional context to understand.

Excerpt 8: Will argues with Derek about Derek's prediction.

1. Derek: [Singing] Hee Haw the (douche) straight like that…straight like that. [Moves B's bar higher than A's bar] That's an "O" "G" call
2. Will: Why you put B? You say that. [points to the bottom row of the probability tree and traces across the outcomes]
3. Derek: I think B is gonna win. And you see all the other times I win…Like with the bunny*—had more opportunity. [*referring to an earlier game in which a rabbit and a horse were competing for points]
4. Will: =The horse, yeah, I know that. I know that. But you shoulda put, on the first one, you said you think A will win because they have more op-por-tun-ity.
5. Derek: No, I didn't say A would win. I think the game is unfair in A's favor, because they have more opportunity=
6. Will: =True that, true that, true that.
7. Derek: Saying douche here. Douche here.
8. Will: Yeah.
9. Derek: Yeah. Mickey Dee's
10. Will: Yeah.
11. Derek: I'm hungry.
12. Will: Me too.
13. Derek: Mickey Dee's in the house.
14. Will: Mickey Dee's in the house.
15. Derek: I be grubbing.
16. Will: Mickey Dee's in my mouth.
17. Derek & Will: [Laughter] [teacher walks by and laughter stops]
18. Derek: [Derek begins typing] Well…Because we…have, how do you spell experience?
19. Will: E-e-x-p-e-r-e-r-i-m-e-n-t. Experiment.
20. Derek: Because we have experimented.
21. Will: Is that how you spell it? [directed to the teacher, who is walking by]
22. Teacher: Perfect. So B's winning?
23. Will: Yeah, on the Try thing. Well, Derek was making B win, but
24. Derek: Don't even worry about that. What you wanna do here, you gonna agree?
25. Will: I'm gonna disagree. [moves agreement bar to disagree]
26. Derek: [Typed: We think Team B will score more points because we have experimented with it all ready.]

Will is apparently applying the normative practice of counting outcomes that was introduced in the previous whole-class discussion. In the previous interaction

(Excerpt 7, turns 2–6), he has visibly counted these outcomes and stated his expectation that team A will win more often because they have more slots.

Derek's interpretation, however, differs in two important ways. First, Derek has interpreted the phrase "in favor of" in the opposite way than his partner. Instead of interpreting the phrase to mean that the game favors one team, he has interpreted it to mean that the unfairness is pointed in the direction of that team. So in turn 5 when he says, "No, I didn't say A would win. I think the game is unfair in A's favor," he means that team A is the recipient of the unfairness and thus less likely to win the game. This misunderstanding points to the importance of the written texts that are resources for the students to understand the games and each other. It also reveals the active, and sometimes unpredictable, construction of meaning that is going on around every phrase.

The second point that the two students disagree about is their interpretation of the ramifications of the unequally partitioned outcome space. Note that both students are aware that the distribution of outcomes is unequally distributed between the two teams. Furthermore, they both agree that this makes the game unfair. The point of contention is that Derek thinks the team with fewer outcomes will win the game, whereas Will believes the opposite is true. To understand this, as Derek does in turn 3, I present some of their shared history with an earlier PIE game. In their first PIE activity, the Horse and Bunny Game, they experimented with the Law of Large Numbers. The horse in this game had fewer ways to score a point than the bunny, but the horse still won the game. Derek is remembering this experience with the first game when he says, "And you see all the other times I win…Like with the bunny— had more opportunity." In turn 18, Derek's reference to "experience" is indexing their previous activity as evidence supporting his current claim.

The Horse and Bunny Game involves analyzing a race between two animals to see if the rules are fair to both participants. The race's outcome is determined by the flip of a coin but not in any simple way. One hundred coins are flipped one at a time. If the coin lands on heads, it is placed on the left side of a balance scale. If it lands on tails, the coin is placed on the right side of the balance scale. The horse moves whenever the scale is balanced (defined as when the percentage of coins flipped so far is between 40% and 60% heads). The bunny moves whenever the scale is unbalanced. Whoever is ahead after the 100 flips wins the race. This game is not a fair game from the perspective of normative probability theory. Over a large number of coin flips, the actual percentage of heads will approximate the theoretical probability of a coin flip (i.e., 50%). Therefore, in most cases, after a few flips the horse will make the majority of the moves.

Derek is asserting that the scoring zones (i.e., the respective ranges when one animal or the other moves) from the Horse and Bunny Game are analogous to the outcomes in the Three-Coin Game, and that even though the bunny had more scoring zones (i.e., a larger range—from 0% to 40% and from 60% to 100% heads), the horse still won in the end. However, this analogy breaks down for two reasons: (a) in the Horse and Bunny Game each scoring zone represents a large number of outcomes, not a single outcome; and (b) each scoring zone is not equally likely, whereas each outcome in the Three-Coin Game is equally likely.

Will seems to agree with Derek for much of this exchange, and he even helps construct their typed response. However, when the teacher intervenes to ask them about their answer (turn 22), Will states that he does not believe their jointly constructed prediction. Here, Will misinterprets Derek's reference about experimenting to refer to their recent experimenting in the Try mode of the Three-Coin Game instead of the Horse and Bunny Game as Derek intended. In turn 23 Will says, "Well, Derek was making B win." He then explicitly states that he disagrees with their response (turn 25), "I'm gonna disagree."

What is important about this initial exchange is that their predictions about the game set the stage for a mathematical argument. This disagreement establishes the context for the students to run the simulation and (potentially) rectify their disagreements. Derek and Will use the data being generated by their simulation of the Three-Coin Game to continue a dispute over their predictions for the game.

As the simulation produces and displays the data that are relevant to the dispute, Derek and Will reinitiate the conflict (see Excerpt 9). Derek attempts to salvage part of his prediction so that he does not lose the dispute. However, Will recreates Derek's full prediction and juxtaposes it with the data from the simulation in an attempt to settle the dispute.

Excerpt 9: Will and Derek renew their argument in the context of empirical data.

1. *Computer:* *At 200 turns team A is ahead.* [computer plays Team A's animation]
2. Will: A be whooping! That's all I am gonna say.
3. *Computer:* *Do you think the game is fair or unfair now?*
4. Derek: Up yours, team A.
5. Will: What you think is this game?
6. Derek: This game is bullshit. …Well my prediction was right, this is game isn't—unfair. [starts new game]
7. *Computer:* *At 20 turns team A is ahead.* [computer stops and displays point totals]
8. Will: Well my prediction is right cause, see at least mine was somewhere close to that prediction. You know what I am saying? The real prediction. But yours is like, umm, totally off because team A is way up there and team B is right there. I am like, "What's up with your prediction?" You know what I am saying? [Will uses his fingers to recreate the bar chart that Derek produced as a prediction to show team B scoring more points. Will then contrasts

this with the bar chart produced by the computer which showed the opposite trend]

9. Derek: You didn't say, you didn't say that team A was gonna win, you said, "yeah that's right"=
10. Will: = I did, I did say team A=
11. Derek: =but didn't you agree with me? =
12. Will: = I said team A, I said team A.
13. Derek: Will, did you or did you not agree with me?
14. Will: I said team A. I said team A. [repeating this in a taunting manner]
15. Derek: Shut your (?) ass.

In this playful, but lively exchange, we see that both Derek and Will agree on the results: Team A wins more of the games (turns 2 and 4). However, in turn 6, Derek attempts to salvage his prediction stating, "Well my prediction was right, this game isn't—is unfair." This, in effect, highlights the general statement that the game is unfair and backgrounds Derek's specific prediction of the direction of that unfairness. Will, however, maintains the dispute by recreating the specific aspect of Derek's prediction. In turn 8, Will uses his fingers to recreate the bar chart that Derek had produced as his prediction and contrasts it with the graph that the computer displayed (see Figures 5A and 5B). This action over the display shows a critical way in which the representations are used to create and maintain a dispute. The rest of the exchange merely perpetuates the dispute without resolution.

Of particular note is the logic and structure of Will's argument. Will's argument is an example of *modus tollens*, which is expressed formally as follows:

if *P* then *Q*
not *Q*
therefore not *P*

In this case Will argues that:

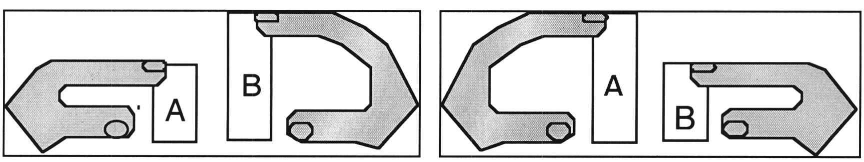

FIGURE 5 (A) Will recreates Derek's predictions using his fingers over the display. (B) Will contrasts Derek's prediction with the actual data by using his fingers to show what the actual bar graph looked like.

if a prediction is "right" (*P*)
then the actual data will "be somewhere close to" the predicted data (*Q*)
Derek's data is "totally off" (*not Q*)
therefore Derek's prediction is not the "real prediction" (*not P*).

Given the importance of logic in succeeding in school and its general association with intelligence in Western culture, this is a bit of impressive reasoning. The structure of Will's argument is equally impressive. This one conversational turn contains a well-structured argument containing a claim, a ground, a warrant, and a backing (Toulmin, 1958). The claim is that Derek's prediction about the game is wrong. The ground for the claim is their shared interpretation of the publicly available bar graph of the actual data. The warrant is the lack of a match between predicted data and actual data visibly produced by Will in front of the computer display. And the backing (i.e., the justification for why the warrant proves the claim) is the aforementioned logic. A substantial amount of the literature in science education discusses the benefits and difficulties in fostering just this kind of a complete scientific argument (e.g., Duschl, 1990; Lemke, 1990; Roth et al., 1999; Wells, 1999).

Perhaps, however, the most impressive and interesting aspect of Will's argument is that his sophisticated reasoning is expressed informally. Rather than seeing their colloquial and colorful language as diminishing the sophistication of the exchange, I see it in a positive light. The form of the talk in this exchange speaks to the fact that the students are engaged with the activity in a meaningful way. They are not parroting some phrase or faithfully following some procedure. They are having a debate, and they are talking in a manner that makes the activity meaningful to them. This type of discourse demonstrates that these students have a relationship with and an understanding of the activity that extends beyond just a schoolish exercise or the compartmentalized circumstances of participating in a laboratory study.

This argument, which spans the three excerpts (7–9) and is sustained for over 15 min, is particularly illustrative because it shows that students' assertions and explanations have social functions as well as cognitive consequences. It was the argument that encouraged the students to make their reasoning explicit and public so that it could be challenged, tested, and modified. Within the context of this dispute they generate justifications of their position by referring to and acting on different sets of representations. Will and Derek make their assertions about who they expect to win the game by manipulating bar charts. Finally, Will was able to settle the dispute by coordinating his gestures with the bar chart display of their data to create a visible proof that Derek's prediction was wrong.

Furthermore, this short example shows the *individual* and *collective* nature of both mathematical talk and representation. It also shows how talk and representation are tightly linked in student interaction. The talk was individual in that the content of the conversation helped change the way each boy individually understood the mathematics of this and future games. Both boys were exposed to new information and ways of interpreting the situation, and both boys were required by their partner to justify and explain their reasoning. This led Derek to revise the way he reasoned about the mathematics of the game. The talk was also collective in that the structures for participation—how they argued, the types of warrants that were accepted, their reactions to challenges—drew on norms established by the classroom community and by the boys' cultural community outside the classroom. For instance, if the classroom was not organized to allow for and promote debate, the interaction just described would have never taken place. In a classroom in which teacher-centered discourse dominated (e.g., the IRE pattern of discourse common in many classrooms), the disagreement may not have become visible; or, if it did, it may have been ignored. However, in this case, the boys' intellectual disagreement was taken up, pursued, and eventually resolved, which directly contributed to their learning.

Evidence of Conceptual Change in Derek

There is some evidence that Derek[7] moved from his initial beliefs of the Two-Coin Game to eventually be attuned to the outcome space as an important resource in deciding if a game is fair—the primary instructional objective for this activity set. This evidence comes later in their investigation, during the *Principles* phase of the activity, when the students were asked to explain how to make a fair game. In this case, Derek and Will make a fair game by equally dividing the eight outcomes between the two teams.

At first, Derek appears to be playing around. Perhaps in recognition of being incorrect about the game earlier, he rearranges the rules of the game such that team B scores 1 point no matter how the three coins land (see Excerpt 10, turn 12). Based on Derek's nonverbal signals (e.g., his laughter) and his subsequent action of unproblematically producing a fair game, it does not seem likely that Derek thinks his first rule set constitutes a fair game. Will certainly does not think it is a fair game and loudly protests, leading Derek to modify the rule set. Derek independently produces a rule set that has four outcomes for team A and four for team B (see turn 27). This shows a change in the organizational logic of Derek's reasoning—he demonstrates that he equates fairness with an "even amount" of chances. Further evidence for a change in Derek's reasoning can be found in turn 7 when he counts the outcomes that have yet to be assigned. This may indicate that he has appropriated the counting practice and is using it to compare the number of outcomes assigned to team B and the number of outcomes available that could potentially be assigned to team A.

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[7]I refer to only Derek because Will displayed the intended practice from the first interaction.

Excerpt 10: Derek and Will discuss what makes games fair and unfair as they construct a fair version of the Three-Coin game.

1. Derek: Now it is up to you to you to make a fair game. ==Make your own rules by dragging the markers to the blue boxes on the bottom of the tree==. [singing as he from the screen]
2. Will: ==Make your own rules by dragging the markers to the blue boxes on the bottom of the tree.== [singing as he reads from the screen]
3. Derek: Combinations for the teams. yeah, baby, gimme a hundred. [reading "combinations for the teams" from the chart label on the lower right chart] Yes in-dee-dee just the ticket. Lassie come home. [Drags team B's chip to assign HHH & HTH to team B. He then moves a third team B chip towards the bottom row of the tree]
4. Will: Put it on that one right there. [pointing to THH]
5. Derek: Or HTT [moves the chip to HTT and drops it]
6. Will: No, we want that one [finger still over THH] that one be winning right there.
7. Derek: One, two, three, four. [counting the remaining unassigned outcomes displayed across the bottom row of the tree using the mouse to point to each in turn]
8. Will: No, move it to that one right there. [pointing as Derek is moving all the pieces] Hey, brother, that ain't fair [Derek is moving all the pieces to score for Team B]
9. Derek: This is a fair game. We got the fairest…Are you zooming in on our game? [talking to the researcher] I picked the game, I watched it on Sportsfocus. [he sets the rule set to BBBBBBBB, all 8 outcomes score for team B]
10. Will: Stop playing! This boy is not making the game fair! He put it all on B.
11. Derek: Alright, alright.
12–24: [Singing into hairbrush. They return to work when the teacher threatens them with detention]
25. Will: They both have an even amount. [final rule set: BAABBAAB, four outcomes for each team in a symmetrical pattern]
26. Derek: That's what I was about to say. Both teams have an even amount. How do you spell amount? Kenny, how do you spell amount?
27. Derek: [Typed: Our game is fair because both team have an even amount of chance to win a game.]

There is also evidence from other sources that indicate conceptual change in the case of Derek. Table 1 shows a 4-point (20%) increase from the pretest to the posttest. This is a gain of 2 standard deviations. More important, when I examined

Derek's answers to questions that address the outcome space and calculating the probability of an event on the posttest, I saw that he often answered and explained his answers appropriately. There were 14 such questions on the posttest, and he answered 9 of these questions correctly. For instance, when asked what the probability of getting three heads in a row was, he answered correctly and justified his answer by enumerating all eight possible outcomes.

The theoretical importance of this second case is that it has shown how a discourse practice—arguing—was extended across different types of activity (i.e., making predictions and interpreting the data) within a single social configuration (i.e., local problem solving), and how the function and productivity of the discourse practice changed with the configuration. If this interpretation of Derek's progress is correct, then Derek's unassisted competence on this activity (i.e., making a fair game) can be traced directly back to his social interaction and argument with his partner in Excerpts 8 and 9. While making their predictions, Derek and Will's argument served to establish and then elaborate the ways in which their understandings diverged. During the simulation, however, their argument served to help them converge on a single understanding. Most important is that this example traces Derek's successful autonomous performance back to his interaction with Will.

Summary of the Two Case Studies

In the first case study, which followed Rosa and Maria, I examined two social configurations, each of which was associated with different types of activities. Following the students' local activity at the computer, we were able to trace a conceptual shift in their reasoning—a shift from naïve intuitions about randomness and luck to drawing inferences from data. I described their local activity as a form of data modeling aimed at understanding what had happened. This accomplishment was then compared to the interactions and intellectual progress that were accomplished in the public configuration of the whole-class discussion. Here, I borrowed from Bowers et al. (1999) who characterized the shift in reasoning as a shift from data modeling (*models of*) to a generative and prospective mathematical practice of using the outcome space (*model for*). Finally, I showed how the local interactions were transformed into opportunities for peer teaching and learning. This transformation would not be possible without the social process of forming community norms within the classroom that required both the individual experience with the software and the collective experience in the whole-class discussion.

The second case, Derek and Will expanded this notion of teaching and learning moments. By closely examining the interactional moves within the dyad, I attempted to show a second way that the outcome practice was propagated across the classroom. In this case, it was peer argumentation that played a critical role in helping one student adopt the normative practice.

Evidence That the Case Studies are Typical

My argument is based primarily on case study data, and I am cautious about making sweeping claims that the particular ways in which these activities and settings interacted to contribute to student learning would hold true in other classrooms or instructional domains. The data record, however, does establish that the specific mathematical practice of reasoning from the outcome space was adopted by the class as a community and by the majority of students as individuals.

Although additional research is needed to verify if the contributions of the local investigations and whole-class discussions as presented here hold true in other contexts, in this section I examine the evidence from this study that the learning outcomes of the case study students are typical. To accomplish this, I briefly present two sources of evidence that the practice of using the outcome space did indeed successfully propagate through the classroom. These two sources of evidence are a pretest–posttest analysis of a pen-and-paper probability test and an analysis of the students' final projects.

First, the pretest–posttest analysis shows that the students in the PIE curriculum were able to apply the outcome space on the posttest successfully. Student performance on written pretests and posttests were measured against a comparison group taught by the same teacher. Students in both the PIE classes and comparison classes were given paper-and-pencil pretest and posttests of the probability concepts addressed by the unit. Items on these tests were derived from standardized tests (National Center for Educational Statistics, 1994), suggestions from the NCTM (1981), items from the research literature on probabilistic reasoning (Konold, 1991; Tversky & Kahneman, 1982), and specific items we designed to assess the probability intuitions relevant to the instructional objectives of the PIE. The score for each student was based on his or her performance on the multiple-choice questions (1 point each) combined with their performance on the short-answer questions (1 point each). The short-answer questions were scored correct if the student justified his or her answer with an appropriate mathematical construct. Tests were scored blind by two researchers.

The PIE students significantly outperformed the comparison class. A three-way analyses of variance (ANOVA) was carried out on three between-subject factors on the posttest: Condition (experimental and comparison), Gender (male and female), and Standardized Test Score (split on the median for this sample). This ANOVA revealed a significant main effect of Condition, $F(1, 90) = 9.7$, $p < .01$, and a significant main effect of Standardized Test Score, $F(1, 90) = 45.7$, $p < .01$. There was no main effect of Gender, $F(1, 90) = 1.3$, $p = .25$, and no interactions were found (Vahey et al., 2000). In addition, t tests found no significant differences between the two groups on the pretest, $t(89) = .21$, $p > .5$, but a significant difference on the posttest, $t(97) = 3.4$, $p < .001$ (see Figure 6; Vahey et al., 2000).

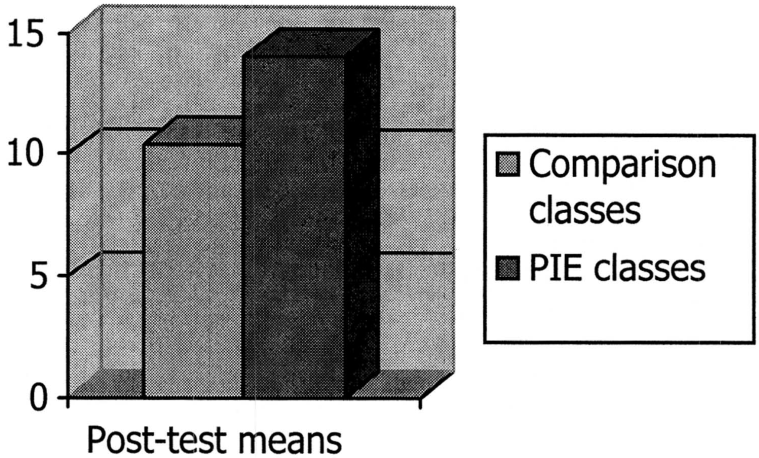

FIGURE 6 Posttest scores.

In addition, the PIE students provided normative explanations to justify their answers more frequently than did the comparison class. For instance, on the posttest, students in the comparison class, even those who were able to calculate the correct answer to a probability question, were more likely to justify their answers with their nonnormative intuitions, such as “it is fair because anything can happen” or “I think that anyone can land, you never know, guess and check.” Students in the PIE group, however, were more likely to justify their answers using the outcome space. For example, when asked if it was more likely to get a mixed outcome when flipping three coins, one student said, “2H and 1T or 2T and 1H might be more likely than the others their chance is six eighths or three fourths.”

Examining the students’ analysis of their own games provides a second source of evidence that the mathematical practice of judging the probability of events based on an analysis of the outcome space did indeed successfully propagate throughout the classroom. An analysis the students’ final projects shows that the majority of students in the PIE group were reasoning using outcome space by the end of the curriculum. For the final project, students were given a number of in-class periods to work with their partner to design a game of chance and write up an analysis that explained why the game was fair or unfair.

Twenty-two of the 23 groups turned in a final project (95%). The games roughly fell into two categories: races and competitions for points. In addition, the games usually relied on a combination of two random devices to determine who moved or scored. The most common random devices were spinners, dice, and coins. Interestingly, most games were kept simple enough that the probabilities could actually be computed theoretically using the techniques the students had developed and practiced. This is somewhat surprising in that it is easy to construct a rule set for a game that quickly makes the probabilities very complicated to compute. This implies the students knew enough about their mathematical practices to create appropriate contexts where they could be applied.

In their final projects, the students often enumerated and partitioned the outcome space to determine the fairness of the game that they created (Tables 2 and 3). Fifteen of the 22 final project write-ups (68%) used some representation of the outcome space to justify their conclusion about the game's fairness. Of these 15 projects, 12 (80%) used the outcome space successfully to come to the mathematically correct conclusion about the game's fairness. In contrast, all 7 of the projects that did not use the outcome space in their analyses failed to correctly analyze their own games (0%). Although the small sample size makes statistical inference problematic, these differences were found to be significant using a Fisher's exact test ($p < .001$). Furthermore, often more than one method was used to reference the outcome space (this explains why the total number of correct conclusions in Table 3 add up to more than 15). Examining the correlation between correctly analyzing the game and the specific representation of the outcome space was also revealing. Again, although the small sample size of the analysis makes these correlations inconclusive, Table 3 suggests that using an equation or a probability tree to represent the outcome space was an important tool in correctly analyzing a game's fairness (based on two-tailed Pearson correlation test, $p < .05$).

The analyses of the pretest and posttest and the students' final projects establish that the majority of students learned some of the basic concepts of probability at a conceptual level deep enough to explain their reasoning and to use the concepts

TABLE 2
Number of Projects That Include the Outcome Space in Their Analysis

Method for Determining Fairness	*Correct Conclusion*	*Incorrect Conclusion*
No mention of the outcome space	0	7
Mentions the outcome space	12	3

TABLE 3
Breakdown of How the Outcome Space was Used in Each Project

Breakdown of Outcome Space Methods	*Correct Conclusion*	*Incorrect Conclusion*
Equation that represented the outcome space*	7	1
Textual description of the outcome space	6	0
List or chart of the outcome space	6	2
Tree diagram of the outcome space*	4	0

*$p < .05$ based on two-tailed Pearson correlation.

productively in new, related contexts. The case studies outline a few of the common paths that students took as they transformed their naïve intuitions into the normative practices. In additional, the close examination of the case studies pull out some of the theoretically relevant interactions that were critical to both the classes' development of the community's mathematical practices and to the individual student's appropriation of these practices.

DISCUSSION AND IMPLICATIONS FOR DESIGN

The major implication of this study is the value of coordinating individual and collective perspectives on learning. In this case, to understand fully how students learned how to reason successfully about games of chance, I investigated how student-learning trajectories were stretched across a number of different activities and social configurations, as well as the ways in which these activities built off one another. In the case of the PIE, there were two complementary social configurations, each with unique forms of participation and socially established goals that contribute to student learning.

In the first configuration, local investigations, the students explored the domain, noticed new relevant features of the environment, and attempted to coordinate these features in new ways to descriptively understand what was going on. The local configuration scaffolded the students' direct experience with probability. The combination of making predictions and modeling the results of the simulations provided important opportunities for knowledge construction and the adoption of new resources for reasoning. Without these experiences, it seems unlikely that the abstractions and mathematical practices developed later would be understood at a conceptual level.

The analyses of the local configuration demonstrate that coordinating individual cognitive processes with the social processes within a community does not entail abandoning the basic principles of constructivist learning theory. A conceptual understanding of probability still is rooted in individuals constructing knowledge from experience (Thompson, 2000). However, in this case, students were attempting to understand both probability and the community's normative ways of interacting with probabilistic situations. Individual students do not invent their own understanding of probability independently, and it is not merely transmitted to them through instruction. The students' experience in pairs, working with the software, directed at understanding the game is contributing to the process by which students construct meaning about why it is sensible to count up outcomes along the bottom row of the probability tree. Furthermore, to the extent that the students begin to adopt the cultural practice of counting, it not only changes their performance, but it transforms how they understand probability at a conceptual level.

In the second configuration, whole-class discussions, the students worked with the teacher to transform their locally constructed ways of reasoning into prospective classroom practices. The classroom discussion was a critical point of transformation, not only because it was where the normative practice of using the outcome space was publicly introduced, but also because it was the configuration that connected the individual to the community. The whole-class discussion served as the place where the findings from students' investigations (in which they were in charge of their own learning) were transformed into cultural norms. How the collective developed this cultural norm (i.e., the practice of using the outcome space) and how it propagated throughout the community shows the importance of giving equal attention to the ways we design our learning environments to support the development of learning communities as we currently do for individuals. Furthermore, these cases demonstrate the value of attending to and designing specifically for the ways that the tools and practices that the community develops are instrumental in shaping the reasoning of individual members. Note that the students were both actively involved in their own learning and actively shaping the practices of their community. It is also worth noting that these two functions were strongly associated with two different social configurations.

What is most interesting, however, is that the interaction between the two social configurations transformed the ways in which the students interacted in the local interactions. By establishing taken-as-shared classroom practices, the students transformed their local investigations from configurations for exploration into configurations for promoting alignment. Although the configuration physically looks the same, the students' participation structures have changed. The roles, responsibilities, and goals change from *constructing a model to make sense of what has happened* to *peer teaching (and learning) to promote alignment within the community.* Accompanying this change are changes in the types of discourse and mathematical practices that are relevant. The very nature and purpose of the configuration as a context for interaction has changed.

This implies that one important consideration for design is to address how this cycle of divergence and convergence can be orchestrated within the classroom. When designing curricula, software, or mathematical activities, it is important to consider how each tool, display, interaction, and activity supports one or both of these two processes. Supporting divergence involves facilitating acts of exploration, imagination, and reflection (Wenger, 1998). This was certainly the case for Derek and Will in their local investigation. Derek, sometimes incorrectly, explored different ways of thinking about the Three-Coin Game and attempted to connect his current activity to his previous activity by noticing features of the current situation that could be mapped back onto his prior investigation. In his argument with Will, we saw how Will coordinated the displays of the data to reflect on the sort of data needed to confirm their predictions. Although this local activity ended with local convergence (cf. Roschelle, 1992), from the perspective of the whole-class there were still many divergent interpretations of the game. These different local models of the game were negotiated in the whole-class discussion leading to convergence and knowledge diffusion (cf. Barab et al., 2000; Roth, 1996).

Another example of convergence was found in the last part of the first case study. Rosa and John's interaction showed how the same discourse practice found in Derek and Will's case (i.e., arguing), emerging from the same social configuration (i.e., a local investigation grounded by the representational structure of the software), took on a dramatically different learning function because of the students' shared history as part of the classroom community. Although previously the local investigations served for the exploration of divergent understandings, after the whole-class discussion, the local investigation served to align the students and promote convergence. This was not a designed feature of the curriculum but a fortuitous and emergent feature from the classroom implementation.

That tools, discourse practices, and activities (designed for specific purposes) can be transformed by ongoing participation as part of a community presents a difficult dilemma for instructional design. It requires us to recognize that tools are not static. The meaning of a tool evolves and is repurposed as part of ongoing joint participation. This implies that one cannot dictate how a piece of software will be used within a classroom. Even if a tool is appropriated as intended, this does not ensure that its use will not change over time. In fact, it is likely that building in tight controls over how tools are used would, in the end, be detrimental to student learning. However, designing tools and curricula that are flexible enough to be successfully adapted and readapted to local communities' evolving purposes requires designers to think in terms of trajectories of learning that span across activities, settings, and time (cf. Cobb, Yackel, & McClain, 2000; Hall & Rubin, 1998).

This tension between personal agency and the appropriation of the normative meanings and practices of existing tools brings me to my point. Within any activity, the cognitive and social aspects of intelligent activity are inseparable. Based on my analyses, I hold that an important mechanism for learning in the PIE environment was the process of providing public accounts to others and that the nature and value of these accounts differ across different social configurations. Students learned probability by learning how to interpret and coordinate the resources provided by the PIE software. One could see this process as occurring within an individual. However, the reason an individual attended to these resources was often social—so she could articulate, explain, and defend a claim. Furthermore, the social goals that were driving interaction changed as a function of the students' understanding of their context.

In the PIE, two critical types of student activity driven by both cognitive and social goals were (a) modeling the data produced by the simulation and (b) producing a mathematical practice that could be used to evaluate these models. Both of these activities encouraged students to participate in particular ways. In addition, both of these activities were associated with different social configurations—modeling data with the local investigations and producing a practice with the public whole-class discussions, respectively.

There are many factors that encourage students to engage in these kinds of modeling activities. One factor identified in this article is that models often originate as warrants to arguments. Sometimes these arguments are between students, as was the case for Derek and Will. Sometimes these arguments are between the students and the computer, as was the case with Rosa and Maria. In either case, the models that students produce to back up their claims serve to make their reasoning visible and allow their reasoning to become an object of discussion and reflection. From the perspectives of the students, one of the likely reasons for these models to be developed into community norms for mathematical practice is that taken-as-shared practices usually require no further explanation (Bowers et al., 1999).

Formal or scientific argumentation is a learned discourse practice in which the interlocutors adopt the conversational roles of rejecting the other's position and providing grounds for their own. During an argument, these roles often switch back and forth depending on whose assertion is being contested in the prior turn. Providing grounds for one's position typically requires some type of explanation or justification that anticipates the recipient's reaction (Toulmin, 1958). That is, the interlocutors are constantly projecting the conversation into the future during the construction of their own turn. When a speaker fails to provide an explanation for an assertion that violates the other participants' expectations, the speaker is often prompted to provide one (Orsolini, 1993). Because of their shared history of production and presumed intersubjective meaning, when students justify an assertion by reproducing a recognizable practice, the assertion is usually not challenged further and is taken as self-evident by the classroom community (Bowers et al., 1999).

At their root, then, classroom mathematical practices are developed, in part, for the social or communicative purpose of settling disputes and not purely for their rational or cognitive value to individuals. That mathematical understandings develop, in part, out of the rhetoric and discourse used to solve social disputes has a strong resonance with the claims made by Latour (1987, 1990) based on his analysis of the professional practice of scientists. Latour argued that the discipline of science is shaped by the ways scientists make and defend knowledge claims. As was the case for the students of the PIE, Latour (1987, 1990) went on to assert that making these claims is not merely a matter of rational and logical evidence, but it also depends on the networks of social and material resources that scientists can tie together to convince others.

On the other hand, I could have reasonably analyzed these interactions from a more traditional psychological perspective and taken the individual student as the unit of analysis. From this perspective, the value of peer interaction would be seen as a way of promoting cognitive conflict within each individual. This cognitive conflict would, in turn, lead an individual to reflect about his or her current understandings, and one would hope that it would also prompt the individual to attempt to reach a new equilibrium (Piaget, 1983). This is, in fact, part of the story.

On the other hand, the perspective that I am trying to develop is that the cognitive aspects of intelligent activity cannot be examined independently from the social aspects. In my analyses, peer interaction did indeed often lead to productive conflicts. However, the resolution and value of these conflicts were both intrapersonal and interpersonal. A purely cognitive analysis would miss that the mechanisms for resolution are social, and the goal of the participants is often social—not cognitive—equilibrium. These case studies highlight that even in formal schooling there are a diverse set of social structures and communicative functions within and around which mathematics gets produced (Greeno & Hall, 1997; Hall, 1995). It is in the social formation of formal knowledge, such as this, where culture and cognition come into contact and create one another (Cole, 1985).

A case in point is the role of the class discussion. Just how important the class discussion was to the process of learning in the PIE curriculum was a surprise to me. As an instructional designer, I had focussed primarily on structuring the software to provide a rich experience and to provide a useful set of tools. My initial expectation was that the students would appropriate the abstraction of the outcome space through their interactions with the software—the *Principles* section of the software was aimed directly at helping students identify and understand these abstractions. However, in both classes we studied, the construct of the outcome space was initially raised, discussed, and elaborated only in the whole-class discussion. It is possible that the instructional moves that we embedded into the software were merely inadequate to achieve this specific shift. However, it is my conjecture that the social goals of creating taken-as-shared practices are inseparable from the cognitive change that eventually was achieved. If this is true, merely replicating in the software the moves that the teacher made in the whole-class discussion would be insufficient because they would not set up the same social need for the collective practice. This seems to warrant further study. It may be that there are some aspects of learning that cannot be effectively addressed with software.

If we are to capitalize on the relationship between individual and collective processes of learning and development, we must design our tools and activities in ways that account for both individual processes of knowledge construction and, simultaneously, social processes by which participation in cultural practices creates a meaningful context that contributes to individual activity. However, the dominant perspectives in educational research have tended to focus on one process at the expense of the other. Researchers tend to privilege either individuals or culture.

As I attempted to demonstrate in this article, it is productive to consider how individual and collective processes interact and co-evolve in the process of human learning. To do so, I had to examine not only students' learning, but also students' learning trajectories. By extending my analysis beyond a single activity or social configuration, I was able to trace at least one of the points of interaction between local and public activity—the development of and alignment with community practices.

As research continues to address learning with all its complexities and interconnections intact, it is increasingly important to consider how the diversity of our theoretical perspectives and empirical methods can be best leveraged to inform educational practice. Although two case studies are clearly insufficient to settle all the claims and issues raised in this article, they serve to illustrate and call attention to a set of issues at the intersection of individual and collective processes of development that have yet to be fully explored. It remains to be seen how far the interplay between social configurations generalizes to other domains and other communities. However, it is at this intersection of the individual and the community that I believe we will begin to make large strides toward a perspective on learning and development that is better suited to inform instruction without ignoring the richness and complexity of the process.

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